PARAMETRIC EQUATIONS FOR ESTIMATING THE CHARACTERISTICS OF THE WORKING MEDIUM OF A CO GASDYNAMIC LASER

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UDC 621.375.82

Calculating the amplification factor of the working medium of a CO gasdynamic laser (GDL) is a complicated task. Usually one must analyze a system of more than 20 differential equations describing the detailed kinetics of V−T and V−V processes together with the equations of gasdynamics [1-3].

In the present article we establish the parametric equations which allow one to calculate the maximum amplification factor of the working medium of a CO gasdynamic laser and the distance at which it is reached, starting from two initial parameters of the system, the frozen-in vibrational energy of the carbon monoxide molecules and the translational temperature of the gas in the resonator region.

Let us consider the escape of the working medium of a CO GDL from an expanding supersonic nozzle which changes, at a given expansion ratio S/S_* of the gas stream, into a channel of constant cross section. The freezing in of the vibrational energy of the molecules occurs as the gas flows near the critical cross section of the nozzle, while the formation of a non-Boltzmann distribution of carbon monoxide molecules over vibrational levels resulting in the formation of a partial population inversion of CO molecules occurs in the plane-parallel section of the nozzle. The maximum amplification factor α of the medium is reached at a certain distance L from the start of the plane section of the nozzle when a quasiequilibrium distribution of molecules is achieved with respect to those vibrational levels for which the amplification factor is highest. In this case

$$\alpha = \psi_{\mathrm{CO}}\, p\Phi(E,\ T,\ \psi_i)H(a), \tag{1}$$

where ψ_i are the molar concentrations of the components of the mixture (i = CO, N_2, Ar, H_2, H_2O, ...); T and p, gas temperature and pressure in the plane section of the nozzle; $a = \Delta_c/\Delta_d = f(T, p)$, ratio of the collisional width Δ_c of the spectral line to the Doppler width Δ_d of the spectral line; E, supply of specific vibrational energy per carbon monoxide molecule; $\Phi(E, T, \psi_i)$, a function determining the maximum population inversion of CO molecules for a given supply of vibrational energy E and translational temperature T of the gas; $H(a)$, Voigt function [4].

It follows from (1) that the quantity $\alpha/(\psi_{\mathrm{CO}}pH(a))$ is a function of the parameters E, T, and ψ_i. As calculations show, the maximum amplification factor of the medium is reached for vibrational levels which are next to the level determining the minimum of the Treanor distribution function of the molecules and far from levels where V−T relaxation becomes important. For this reason $\Phi(E, T, \psi_i)$ is a weak function of the mixture and has the form

$$\alpha/(\psi_{\mathrm{CO}}\, pH(a)) = f_0(E,\ T). \tag{2}$$

For nitrogen-free working media of CO GDL the quantity E is practically equal to the frozen-in vibrational energy of the carbon monoxide molecules. For mixtures containing nitrogen in a molar concentration ψ_{N_2} the quantity E can be found approximately from an expression obtained using Teare's equation [5],

$$\left(1 + \frac{\psi_{N_2}\Theta_{N_2}}{E_0 - \psi_{\mathrm{CO}}E}\right)\Big/\left(1 + \frac{\Theta_{\mathrm{CO}}}{E}\right) = \exp\left(\frac{\Theta_{N_2} - \Theta_{\mathrm{CO}}}{T}\right), \tag{3}$$

where Θ_{N_2} and Θ_{CO} are the characteristic temperatures of nitrogen and carbon monoxide molecules, respectively; E_0 is the average, specific, frozen-in vibrational energy per molecule of the N_2 + CO mixture.

The quantity E_0 can be found from the solution of the equation of relaxation of vibrational energy of a gas in an expanding supersonic stream or approximately using the semiempirical expression

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Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 22-27, May-June, 1982. Original article submitted April 3, 1981.

TABLE 1

Variant number	T_*, °K	p_*, Pa	ψ_{CO}	ψ_{H_2}	p, Pa	E, °K	$\alpha/(\psi_{CO}pH(\alpha))$, $(m\cdot Pa)^{-1}$	$\psi_{CO}pL$, cm·Pa
1	2000	$4\cdot10^6$	0,2	0,1	$8,7\cdot10^2$	700	$2,2\cdot10^{-2}$	$1,5\cdot10^4$
2	2000	$4\cdot10^6$	0,2	0,1	$8,7\cdot10^2$	530	$9\cdot10^{-3}$	$1,6\cdot10^4$
3	2000	$4\cdot10^6$	0,2	0,001	$1,2\cdot10^3$	840	$3,4\cdot10^{-2}$	$1,2\cdot10^4$
4	2000	$4\cdot10^6$	0,2	0,1	$8,8\cdot10^2$	740	$2,5\cdot10^{-2}$	$8,3\cdot10^3$
5	2000	$4\cdot10^6$	0,2	0,005	$1,2\cdot10^3$	830	$3,2\cdot10^{-2}$	$1,2\cdot10^4$
6	2000	$4\cdot10^6$	0,2	0,2	$6,3\cdot10^2$	610	$1,5\cdot10^{-2}$	$1,8\cdot10^4$
7	1888	$4\cdot10^6$	0,895	0,105	$1,2\cdot10^2$	630	$1,8\cdot10^{-2}$	$4,8\cdot10^4$
8	1800	$4\cdot10^6$	0,4	0,1	$6,4\cdot10^2$	560	$1,4\cdot10^{-2}$	$3,9\cdot10^3$
9	2000	$1,5\cdot10^6$	0,2	0,1	$3,3\cdot10^2$	810	$3,5\cdot10^{-2}$	—
10	2000	$1,5\cdot10^6$	0,2	0,1	$3,3\cdot10^2$	820	$3,6\cdot10^{-2}$	—
11	2000	$1,5\cdot10^6$	0,2	0,1	$3,5\cdot10^2$	730	$2,6\cdot10^{-2}$	—
12	2000	10^7	0,2	0	$3,1\cdot10^3$	850	$3,1\cdot10^{-2}$	$4,7\cdot10^3$
13	2000	10^7	1	0	$2,8\cdot10^2$	850	$3,1\cdot10^{-2}$	$6,8\cdot10^3$
14	3000	10^7	0,2	0	$1,1\cdot10^3$	1740	$1,1\cdot10^{-1}$	$2,8\cdot10^3$
15	3000	10^7	0,4	0	$5,2\cdot10^2$	1740	$1,3\cdot10^{-1}$	$1,6\cdot10^3$
16	3000	10^7	0,6	0	$2,7\cdot10^2$	1740	$1,3\cdot10^{-1}$	$1,7\cdot10^3$
17	3000	10^7	1	0	$6,7\cdot10^1$	1740	$1,2\cdot10^{-1}$	$1,5\cdot10^3$

TABLE 2

Variant number	T_*, °K	p_*, Pa	ψ_{CO}	T, °K	p, Pa	E, °K	$\alpha/(\psi_{CO}pH(\alpha))$, $(m\cdot Pa)^{-1}$	$\psi_{CO}pL$, cm·Pa
1	2400	$1,5\cdot10^6$	0,2	50	$2,9\cdot10^1$	1200	$9,1\cdot10^{-1}$	$9,3\cdot10^2$
2	1600	$1,5\cdot10^6$	0,2	50	$9,5\cdot10^1$	530	$3\cdot10^{-1}$	$6,2\cdot10^3$
3	1600	$4\cdot10^6$	0,124	50	$3,1\cdot10^2$	470	$3,7\cdot10^{-1}$	$6,4\cdot10^3$
4	2000	$4\cdot10^6$	0,2	50	$1,2\cdot10^2$	700	$4\cdot10^{-1}$	$2\cdot10^3$
5	1600	$1,5\cdot10^6$	0,2	75	$2,8\cdot10^2$	530	$5,3\cdot10^{-2}$	$1,8\cdot10^4$
6	1800	$1,5\cdot10^6$	0,2	75	$2,8\cdot10^2$	680	$8,5\cdot10^{-2}$	—

$$T_*/T_V = 1 + 0{,}027\omega^{0{,}6}, \tag{4}$$

for the frozen-in vibrational temperature T_V [6], where $\omega = \tau_r/\tau_s$ is the ratio of the time of relaxation τ_r of vibrational energy at the gas temperature T_* and pressure p_* in the critical cross section of the nozzle to the characteristic time of stream expansion τ_s in the initial section of the nozzle.

The analysis of a simplified CO GDL model also allows one to find the connection between the distance L at which the amplification factor of the medium in the plane-parallel section of the nozzle reaches a maximum and the parameters E and T of the working mixture.

When a non-Boltzmann distribution of CO molecules over vibrational levels is formed in the plane section of the nozzle the maximum amplification factor of the medium is reached for a quasistationary distribution of molecules over vibrational levels in the region of the minimum of the Treanor distribution function. The characteristic time of establishment of such a distribution for nitrogen-free mixtures is determined mainly by V–V exchange between CO molecules which are at the vibrational level v_* corresponding to the minimum of the Treanor function and molecules which are at the first level, where a large part of the vibrational energy is concentrated. The variation of the population of the v_* level with time can be represented approximately through a certain function $F(n_{v_*}, n_1, T)$ describing the kinetics of V–V exchange,

$$u\frac{dn_{v_*}}{dx} = \psi_{CO}pF(n_{v_*}, n_1, T) \approx \psi_{CO}pF(n_{v_*}, E, T), \tag{5}$$

since $n_1 = n_1(E)$. Here u is the velocity of the gas stream; n_1 and n_{v_*} are the relative populations of the first and v_*-th vibrational levels of the carbon monoxide molecules, respectively; x is the current length.

The solution of Eq. (5) with the initial condition $n_{v_*} \approx 0$ has the form

$$n_{v_*} = f_1(\psi_{CO}px/u, E, T). \tag{6}$$

TABLE 3

Variant number	T_*, °K	ψ_{CO}	T, °K	p_*, Pa	E, °K	$\alpha/(\psi_{CO}pH(\alpha))$, $(m \cdot Pa)^{-1}$	$\psi_{CO}pL$, cm · Pa
1	2000	0,2	75	$1,4\cdot10^{3}$	850	$9,2\cdot10^{-2}$	$1,6\cdot10^{3}$
2	2000	1	75	$1,1\cdot10^{2}$	850	$1,1\cdot10^{-1}$	$2,5\cdot10^{3}$
3	3000	0,2	75	$4,8\cdot10^{2}$	1740	$3,6\cdot10^{-1}$	$7,5\cdot10^{2}$
4	3000	0,4	75	$2,3\cdot10^{2}$	1740	$3,6\cdot10^{-1}$	$6,7\cdot10^{2}$
5	3000	0,6	75	$1,1\cdot10^{2}$	1740	$3,6\cdot10^{-1}$	$7,3\cdot10^{2}$
6	3000	1	75	$2,7\cdot10^{1}$	1740	$3,4\cdot10^{-1}$	$6,9\cdot10^{2}$
7	2000	0,2	50	$4,8\cdot10^{2}$	850	$4,8\cdot10^{-1}$	$4\cdot10^{2}$
8	2000	1	50	$2,7\cdot10^{1}$	850	$5,3\cdot10^{-1}$	$6,7\cdot10^{2}$
9	3000	0,2	50	$1,6\cdot10^{2}$	1740	1,2	$2,3\cdot10^{2}$
10	3000	0,4	50	$6,7\cdot10^{1}$	1740	1,3	$2,4\cdot10^{2}$
11	3000	0,6	50	$2,7\cdot10^{1}$	1740	1,4	$2,8\cdot10^{2}$
12	3000	1	50	6,7	1740	1,5	$2,4\cdot10^{2}$
13	3000	0,2	30	$4\cdot10^{1}$	1740	9,5	$1,1\cdot10^{2}$
14	3000	0,4	30	$1,3\cdot10^{1}$	1740	7,9	$1,1\cdot10^{2}$
15	3000	0,6	30	6,7	1740	8,7	$1,1\cdot10^{2}$
16	3000	0,8	30	1,1	1740	9,7	$1,3\cdot10^{2}$

When a quasistationary distribution of CO molecules over vibrational levels is reached at a distance x = L, $f_1'(\psi_{CO}px/u, E, T)|_{x=L} = 0$ and hence $\psi_{CO}pL/u = f_2(E, T)$. Since u is a slowly varying function in all the variants of CO GDL calculations of practical interest, we find that the quantity $\psi_{CO}pL$ is a function of the parameteters E and T,

$$\psi_{CO}\, pL = f_3(E,\ T). \tag{7}$$

A similar relation can be obtained for nitrogen-containing mixtures with $\psi_{N_2} \gg \psi_{CO}$:

$$\psi_{N_2}pL = f_4(E_0 - \psi_{CO}E, T). \tag{8}$$

The concrete form of the functions (2), (7), and (8) can be found by numerical calculations of individual CO GDL variants.

In the present work we carried out a numerical experiment with working mixtures of a CO gasdynamic laser containing CO, H_2, Ar, and H_2O in varying concentrations for different initial parameters T_* and p_* of the mixture and final temperatures of the expanded gas stream. In the general case we solved a system of gasdynamic equations for a working mixture escaping from a supersonic nozzle together with the kinetic equations [1-3] describing V−V and V−T exchange between the lower 26 vibrational levels of the CO molecule, as well as the deactivation of vibrationally excited carbon monoxide molecules on H_2, Ar, H_2O, and hydrogen atoms. The rate constants of the V−V and V−T processes were calculated from the equations of [1-3, 7], drawing upon the known experimental data on relaxation times.

The calculation was carried out in a wide range of variation of the GDL parameters p_*, T_*, S/S_*, and φ and the concentrations of the components of the mixture (φ is the initial half-angle of the nozzle aperture): $1.5 \cdot 10^6 \le p_* \le 10^7$ Pa, $1800 \le T_* \le 3000$°K, $100 \le S/S_* \le 15{,}000$, $0.5 \le \tan\varphi \le \infty$. The corresponding values of the stagnation temperature and pressure T_0 and p_0 are $T_0 = T_*(\gamma + 1)/2$ and $p_0 = p_*(T_0/T_*)^{\gamma/(\gamma-1)}$, where γ is the effective adiabatic index of the mixture. For $\gamma = 1.4$, $T_0 = 1.2\, T_*$, and $p_0 = 1.89\, p_*$. The hydrogen concentration ψ_{H_2} in the mixture was varied from zero to 30% while the water vapor concentration was varied from zero to 10%.

The main results of the calculation are presented in Tables 1-3. The height of the critical cross section of the nozzle was chosen as 0.3 mm and the half-angle φ was 28° for $p_* = 4 \cdot 10^6$ Pa and 45° for $p_* = 1.5 \cdot 10^6$ Pa. For hydrogen-free mixtures with $p_* = 10^7$ Pa we used a model of instantaneous expansion of the mixture. In Table 1 we present the results of a calculation of the parameters of the working medium of a CO GDL for T = 100°K and different initial values of p_* and T_* and concentrations of the components CO, Ar, and H_2 in the mixture. Variant 2 corresponds to a calculation for a mixture containing an admixture of 0.5% H_2O and variant 11 to 1% H_2O. In variant 4 we ignored the relaxation of CO molecules on hydrogen atoms, while in variant 9 the constant for relaxation of CO molecules on hydrogen atoms was doubled. The results of a calculation with $\psi_{H_2} = 0.1$ and $\psi_{H_2O} = 0$ and different values of p_*, T_*, T, and ψ_{CO} are presented in Table 2, while the results for a hydrogen-free CO−Ar mixture with $p_* = 10^7$ Pa are presented in Table 3.

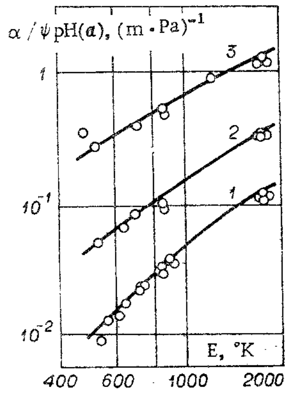

Fig. 1

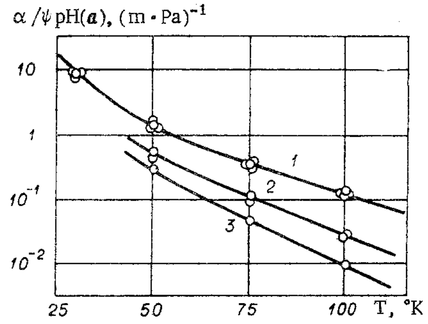

Fig. 2

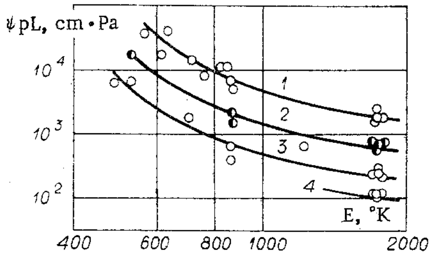

Fig. 3

The calculated values of the parameter $\alpha/(\psi_{CO}pH(\alpha))$ for different values of the energy E at a fixed translational gas temperature T = 100, 75, and 50°K (Tables 1-3) are presented in Fig. 1 (curves 1-3, respectively). The results of the numerical experiment for each value of T are well described by one universal curve. With an increase in the frozen-in vibrational energy of the CO molecules the value of the parameter $\alpha/(\psi_{CO} \cdot pH(\alpha))$ increases sharply. Analogous dependences also occur for other values of the temperature T. For a fixed energy E, $\alpha/(\psi_{CO}pH(\alpha))$, grows with a decrease in T (Fig. 2, E = 1740, 850, and 530°K for curves 1-3, respectively).

The quantity $\psi_{CO}pL$, which determines the distance at which the amplification factor of the medium reaches the maximum value, essentially depends on the value of the initial parameter E (Fig. 3, T = 100, 75, 50, and 30°K for curves 1-4, respectively), decreasing as E increases at a fixed temperature T. For a given value of E the parameter $\psi_{CO}pL$ is the larger, the higher T.

In order to estimate the maximum value of the amplification factor of the working medium of a gasdynamic laser and the corresponding nozzle length L for any set of initial CO GDL parameters using Figs. 1-3, one must first find the values of p and T in the region of the GDL resonator from gasdynamic calculations and determine the value of the initial parameter E by a numerical calculation or from the empirical equation (4).

The use of the parametric equations offered in the present article can be helpful for a rapid estimate of the characteristics of the working medium of a CO GDL with arbitrary parameters.

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